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13. ABSTRACT (Maximum 200 words) Several fast signal processing algorithms applicable in target detection and recognition techniques have been developed. These include a novel accurate and efficient method for replacing IIR filters by FIR filters, a method for multiresolution implementation of operators via filter banks, a multiresolution strategy for numerical homogenization and an algorithm for Unequally Spaced Fast Fourier Transform (USFFT) in multiple dimensions. The USFFT algorithm was one of the specific goals of the proposal. This algorithm was shown to be about 20% more efficient than the previous approach. It was also used as a part of a code for computing antenna far field patterns from near field measurements.				
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## **FINAL REPORT**

### **FAST NUMERICAL ALGORITHMS AND SAR APPLICATIONS**

**ARPA/AFOSR Grant F49620-93-1-0474**

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## ORIGINAL OBJECTIVES:

- (1) the development of fast numerical algorithms which have general applicability in target detection and recognition techniques*
- (2) the development of fast and accurate algorithms for SAR which are generalizable for a multistatic SAR configuration*

## RESULTS:

Several fast signal processing algorithms applicable in target detection and recognition techniques have been developed. These include a novel accurate and efficient method for replacing IIR filters by FIR filters, a method for multiresolution implementation of operators via filter banks, a multiresolution strategy for numerical homogenization and an algorithm for Unequally Spaced Fast Fourier Transform (USFFT) in multiple dimensions. The USFFT algorithm was one of the specific goals of the proposal and as a tool for processing SAR data appears to be about 20% more efficient than using the interpolation approach with Parks-McClellan filter design.

We have developed a multiresolution strategy for homogenization. Homogenization may be defined as an analysis in which we construct equations describing coarse-scale behavior of the solution while ignoring fine-scale detail. We have developed algorithms for the homogenization via multiresolution analysis (MRA) of systems of linear ODE's with variable coefficients and forcing terms, of small systems of non-linear ODE's and of strictly elliptic linear PDE's. For elliptic PDE's our approach also provides a direct solver. The work in this direction is continuing and there is confidence that our approach can be extended for problems of wave propagation and signal processing.

An algorithm for multiresolution segmentation of SAR images has been developed and preliminary tests were performed on SAR data. Theoretical approaches to processing SAR data collected along curved paths and bistatic configurations have been developed. It appears that a Fast Fourier Transform along curves (1D) and surfaces (2D) (further generalization of USFFT) plays an important role. An approach to inverse USFFT has been developed. The work in this direction is continuing.

## ACCOMPLISHMENTS:

We have developed several algorithms applicable in signal processing and, specifically, processing of SAR data. The following is a brief description of these algorithms.

### *1. A novel accurate and efficient method for replacing IIR filters by FIR filters*

In finding FIR approximations of IIR filters, the standard approach consists in using some optimization criterion (e.g. least squares) for a fixed length of FIR filter. The reason for fixing the degree of approximating polynomial is to ensure efficiency of the resulting filter. Departing from the traditional approach, we attain efficiency by considering a factored FIR approximation with small number factors where implementation of each factor is inexpensive in spite of the fact that the degree of approximating polynomial might be large. As a result, we may achieve an accurate and efficient implementation which may be viewed as an alternative to structurally passive IIR implementations. In the case of non-causal IIR filters, our approach allows one to improve accuracy of the output adaptively as more data become available.

*Significance:* This is a technical result with important implications for implementation of IIR filters. We expect a wide use of this approach in software and hardware implementations.

### *2. A method for multiresolution implementation of operators via filter banks*

We introduce a method for implementation of operators via filter banks in the framework of the Multiresolution Analysis. The design of digital filters is always a trade-off between accuracy and efficiency. For a number of operators this trade-off obtained via traditional filter design techniques is not adequate, especially if high precision is required. In our method we decompose a signal into different scales (subbands) and implement operators as subband filters. In particular, we obtain a fast algorithm for the computation of the Hilbert transform, and a pyramidal algorithm for discrete wavelet transform with complex analytic (or progressive) wavelet.

*Significance:* This result allows one to use filter banks not only for compression and coding of signals but also for their processing. Our method yields efficient subband filters and is capable of achieving any given accuracy. Has a strong potential for hardware implementations.

### 3. A multiresolution strategy for numerical homogenization.

Homogenization may be defined as an analysis in which we construct equations describing coarse-scale behavior of the solution while ignoring fine-scale detail. This work concerns a multiresolution strategy for homogenization of differential equations. We consider the homogenization via multiresolution analysis (MRA) of systems of linear ODE's with variable coefficients and forcing terms., of small systems of non-linear ODE's and of strictly elliptic linear PDE's. For elliptic PDE's our approach also provides a direct solver. We develop an efficient numerical approach which generates the coefficients of the homogenized equation. As one of the examples we treat wave propagation in a stratified medium.

*Significance:* This result appears to be the first efficient numerical homogenization technique using MRA as a tool. The results for elliptic PDE's include a proof and a numerical demonstration of the fact that small eigenvalues are preserved under the reduction procedure. These results indicate that the approach should also work for problems of wave propagation. Problems like scattering from rough surfaces may be addressed which will lead to applications in signal processing.

### 4. An algorithm for fast and accurate computation of the Fourier transform of functions with large number of discontinuities and

### 5. An algorithm for Unequally Spaced Fast Fourier Transform (USFFT) in multiple dimensions

We have developed a simple approach for the fast evaluation of the Fourier transform of functions with singularities based on projecting such functions on a subspace of Multiresolution Analysis. We obtain an explicit approximation of the Fourier Transform of generalized functions and develop a fast algorithm based on its evaluation. In particular, we construct an algorithm for the Unequally Spaced Fast Fourier Transform and test its performance in one and two dimensions. The number of operations required by these algorithms is  $O(N \log N + N_p (-\log \epsilon))$  in one dimension and  $O(N^2 \log N + N_p (-\log \epsilon)^2)$  in two dimensions, where  $\epsilon$  is the precision of computation,  $N$  is the number of computed frequencies and  $N_p$  is the number of nodes.

*Significance:* Although there were several algorithms of this type of available in 1D, this one is particularly simple, easily generalizes to 2D and appears to be very efficient. The Fast Fourier Transform (FFT) algorithm requires sampling on an equally spaced grid which proves to be a significant limitation in many applications. The USFFT removes this limitation and we expect it to be widely used in a variety of applications.

The two-dimensional USFFT algorithm is directly applicable to processing of SAR data. Jointly with ERIM (John Gorman), we have evaluated its performance for SAR data processing. It appears to be about 20% more efficient than using the interpolation approach with Parks-McClellan filter design. Besides SAR, there are also applications to X-ray tomography, MRI and Ultrasound Imaging which we are pursuing as well.

*6. An algorithm for multiresolution segmentation of SAR image has been developed and preliminary tests were performed on SAR data.*

*Significance:* An algorithm of this type may be used for ATD/R since it permits the introduction of "decision making" trees, finding bright reflectors, compression of the data, etc. We have implemented a "research" version of this algorithm.

*7. Fast Fourier Transform over curves and surfaces (still in progress).*

*Significance:* A successful algorithm of this type appears to permit processing of SAR data collected along irregular paths and via bistatic configurations. We also found applications of such transform to fast evaluation of oscillatory matrices on vectors.

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7. G. Beylkin and N. Coult, *A Multiresolution strategy for reduction of elliptic PDE's and eigenvalue problems*, PAM Report 270, March 1996, submitted to Applied and Computational Harmonic Analysis.
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**Remark:** As of August 15, 1994, Mary E. Brewster left CU and joint Battelle Pacific Northwest Lab. Supported again for one month during the summer of 1995.

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**WILLY HEREMAN**, associated with this research effort  
supported during 1993-94 academic year

Visiting Associate Professor

Permanent affiliation: Dept. of Math. and Computer Sciences, Colorado School of Mines

**LUCAS MONZON**, associated with this research effort  
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**TALKS :**

Martin Marietta workshop "Advanced Topics in Wavelets and Adapted Waveform Analysis", October 93

Automatic Target Recognition 93 Conference, Nov 1 - Nov 4, 1993 at MIT Lincoln Labs, Lexington, MA

Conference on multiresolution methods at Technion, Israel, May 1994

Meeting of SPIE, San-Diego, July 1994

ARPA Program Review Arlington, VA, August 3-5, 1994

ERIM, Ann Arbor, MI, Oct. 1994

IMA Tutorial and 2 lectures at the Workshop, Minneapolis, Fall 1994

Battelle Pacific Northwest Lab, Jan. 1995.

ARPA Program Review Arlington, VA, May-June 1995.

Princeton University, May 1995.

AMS-SIAM Summer Seminar, Park City, Utah, Aug. 1995.

Universite de Montreal, March 1996

Los Alamos National Lab, April 1996

Battelle Pacific Northwest Lab, June. 1996.

**NEW DISCOVERIES, INVENTIONS, OR PATENT DISCLOSURES:**

None.

**TECH TRANSFERS:**

1. As a part of Industry/University Automatic Target Detection, Recognition, and Identification Initiative, we are involved (since August 1994) in the project

**Efficient SAR Processing via Multiresolution Wavelet Techniques**

as a joint effort with Environmental Research Institute of Michigan (ERIM)

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2. The code for USFFT has been provided to Brad Alpert at National Institute of Standards and Technology in Boulder. Brad Alpert has implemented a code for computing antenna far field patterns from near field measurements. This is an important application of USFFT which extends scanning technology to higher frequencies and mobile platforms.